Section P.3: Radical and Rational Exponents

if $b^2 = a$ then $\sqrt{a} = b$

- very helpful to know perfect square and cube numbers.

Properties:

$\sqrt{a^2} = |a|$ $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{(-3)^2} = 3$

$\sqrt{9}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$a^{\frac{1}{2}} = \sqrt{a}$

Example:

$\sqrt{500}$

50 10

2 25 2 5

5 5

$2\cdot5\sqrt{5}$

$10\sqrt{5}$

$\sqrt{6x}\cdot\sqrt{3x}$

$\sqrt{18x^2}$

2 9

3 3

$3\sqrt{2x^2}$

$\boxed{3\cancel{x}\sqrt{2}}$

$\sqrt{x^2}=|x|$

$\sqrt[4]{x^3}$

$\sqrt[4]{x^8}$

x^2

$3\sqrt{2x^2}$

$3\sqrt{2}\,x$

$\sqrt{x^2}=|x|$

Example:

$\frac{\sqrt{100}}{\sqrt{9}} = \frac{10}{3}$

$\sqrt{\frac{100}{9}}$

$\frac{\sqrt{48x^3}}{\sqrt{6x}}$

$2\sqrt{x^2x}$ $\sqrt{x^2}=|x|$

$\frac{4x\sqrt{3x}}{\sqrt{6x}}$

48

2 24

2 12

2 6

2 3

$\frac{4x\sqrt{3x}}{\sqrt{6x}} \cdot \frac{\sqrt{6x}}{\sqrt{6x}}$

$\frac{4x\sqrt{18x^2}}{6x}$

18

2 9

3 3

$\frac{12x^2\sqrt{2}}{6x}$

$\boxed{2x\sqrt{2}}$

- Just like we have like terms we have like radicals....

Example:

$7\sqrt{2}+5\sqrt{2}$

$12\sqrt{2}$

$1\sqrt{5x}-7\sqrt{5x}$

$-6\sqrt{5x}$

5

9

5

Examples:

$7\sqrt{3}+\sqrt{12}$

4 3

2 2

$7\sqrt{3}+2\sqrt{3}$

$9\sqrt{3}$

$4\sqrt{50x}-6\sqrt{32x}$

2 25 2 16

5 5 8 2

4 2

2 2

$20\sqrt{2x}$

$20\sqrt{2x}-24\sqrt{2x}$

$\boxed{-4\sqrt{2x}}$

Rationalizing the Denominator....

- you cannot leave any radicals on the bottom of a fraction.

$$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{3}}{3}$$

$$\frac{1}{2+\sqrt{3}}$$

Example:

$$\frac{15}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}}$$

$$\frac{15\sqrt{6}}{6}=\frac{5\sqrt{6}}{2}$$

$$\frac{12}{\sqrt{8}}$$

2

Rationalizing Larger Denominators

- with use of the conjugate.

$2+\sqrt{3} \rightarrow 2-\sqrt{3}$

$$\frac{7}{(5+\sqrt{7})}\cdot\frac{(5-\sqrt{7})}{(5-\sqrt{7})} = \frac{35-7\sqrt{7}}{18} = \frac{7(5-\sqrt{7})}{18}$$

$25 \cancel{-5\sqrt{7}+5\sqrt{7}} - 7$

$$\frac{1}{\sqrt{3}+2}\cdot\frac{(\sqrt{3}-2)}{(\sqrt{3}-2)} = \frac{\sqrt{3}-2}{-1} = -\sqrt{3}+2 = 2-\sqrt{3}$$

$3-4$

Other Roots and Properties:

$\sqrt[n]{a}=b$ means $b^n=a$

if n is odd then $\sqrt[n]{a^n}=a$

if n is even then $\sqrt[n]{a^n}=|a|$

Example:

$\sqrt[3]{24}$ $\quad \sqrt[4]{8}\bullet\sqrt[4]{4}$ $\quad \sqrt[4]{\frac{81}{16}} = \frac{\sqrt[4]{81}}{\sqrt[4]{16}}$

2 12

4 3

2 2

$2\sqrt[3]{3}$

$\sqrt[4]{8\cdot 4}$

32

4 2 2 2

2 2

$2\sqrt[4]{2}$

81 16

9 9 4 4

3 3 3 3 2 2 2 2

$\frac{3}{2}$

Examples:

$5\sqrt[3]{16} - 11\sqrt[3]{2}$

4 4

2 2 2 2

$10\sqrt[3]{2} - 11\sqrt[3]{2}$

$-1\sqrt[3]{2}$

Rational Exponents

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

inside

out

$$a^{-\frac{1}{n}} = \frac{1}{a^{\frac{1}{n}}} = \frac{1}{\sqrt[n]{a}}$$

$$x^{\frac{3}{2}} = \sqrt{x^3}$$

Examples:

$64^{1/2}$

$\sqrt{64}$

8

$8^{1/3}$

$4 \quad \sqrt[x]{16}$

$64^{-1/3}$

$\frac{1}{\sqrt[3]{64}}$

$\frac{1}{4}$

Examples:

$27^{2/3}$ $9^{3/2}$ $16^{(-3/4)}$

9^(3/2)

27

$\sqrt{9^3}$

$\sqrt{9^2 \cdot 9}$

$9 \cdot 3 = 27$

Examples

$\left(5x^{1/2}\right)\left(7x^{3/4}\right)$

$$\frac{32x^{5/3}}{16x^{3/4}}$$

$2x^{\cancel{5/3} - \cancel{3/4}}$ $\frac{20}{12}$ $\frac{9}{12}$

$2x^{\frac{11}{12}}$

Examples:

$$\sqrt[9]{x^3} = x^{\frac{3}{9}} = x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$\sqrt[8]{x^2} \qquad \sqrt[4]{x}$$

Suggested Homework:
Chapter P.3 pg.32 #'s
9,11,17,21,25,29,31,33,37,
41,43,45,67,73,75,81,87,
91,101